More on Angular Momentum

P221: November 8, 2013
## Summary

**Translational (Linear) Motion** | **Rotational Motion**
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Displacement: $\vec{x}$ | Angular Displacement: $\vec{\theta}$
Velocity: $\vec{v}$ | Angular Velocity: $\vec{\omega}$
Acceleration: $\vec{a}$ | Angular Acceleration: $\vec{\alpha}$
Mass: $m$ | Moment of Inertia: $I$
Force: $\vec{F}$ | Torque: $\vec{\tau}$
Kinetic Energy: $\frac{1}{2}mv^2$ | Rotational Kinetic Energy: $\frac{1}{2}I\omega^2$

**Newton's 2\textsuperscript{nd} Law**
- Linear: $\vec{F}_{\text{Net}} = m\vec{a}$
- Rotational: $\vec{\tau}_{\text{Net}} = I\vec{\alpha}$

**Linear momentum** $\vec{p} = m\vec{v}$ | **Angular momentum** $\vec{L} = I\vec{\omega}$
Clicker Question

A student is riding on the outside edge of a merry-go-round rotating about a frictionless pivot. She holds a heavy ball at rest in her hand. If she releases the ball, the angular velocity of the merry-go-round will:

A) Increase  B) Decrease  C) Stay the same
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Discussion

Total initial angular momentum is from merry-go-round, student, and ball
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\( L \) vector is out of the page

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L_{\text{total}} = L_{m-g-r} + L_{\text{student}} + L_{\text{ball}}
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Are there any external forces?

We learned particle in straight line has same $L$
Clicker Question

A student holding a heavy ball sits on the outer edge a merry-go-round which is initially rotating counterclockwise. Which way should she throw the ball so that she stops the rotation?

• A) To her left
• B) To her right
• C) Radially outward
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Discussion

In the student, ball, and merry-go-round system, are there any external forces?

top view: initial

final
Discussion

In the student, ball, and merry-go-round system, are there any external forces?

→ NO!

Angular momentum $L$ of total system is conserved (constant)

$$L_{\text{total}} = L_{m-g-r} + L_{\text{student}} + L_{\text{ball}} = \text{constant}$$
Discussion

Case C:
Ball is given a force that is parallel to radial distance $\vec{F} \parallel \vec{R}$

Therefore, there is no net torque and no way to add angular momentum to the ball $\Delta L_{\text{ball}} = 0$

Because total $L$ is constant, there is no change in any angular momentum
Discussion

Case B:
What direction is initial angular momentum?
→ Out of the page

What is the ball’s direction of angular momentum?
→ Into the page

BUT the total $L$ is constant. Any angular momentum $\Delta L$ “lost” by the ball MUST be “gained” by the merry-go-round and student

The vector nature of angular moment makes a big difference!
Discussion

Case A:
We want $L_{\text{ball}} = L_{\text{total}}$ this requires $L_{\text{m-g-r}} = L_{\text{student}} = 0$

Again, vector must point in right direction (out-of-page in this case)

Therefore, if merry-go-round and student “lose” angular momentum, and angular momentum is conserved, then ball must “gain” angular momentum in out-of-page direction

To “gain” requires more velocity to the left, and we know a particle in a straight line still carries angular momentum from last lecture
Clicker Question

A tetherball of mass $m$ is spun around a pole of diameter $D$ with a velocity $v$. You draw the cord through a hole in the center of the pole.

After pulling, does the angular momentum $L$ change? How about the kinetic energy?

A.) Both stay the same
B.) Both change
C.) Angular momentum changes, but energy is the same
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Discussion of Tetherball 1

Inward spiraling trajectory

Top view
Discussion of Tetherball 1

External force $F$ is provided by pulling cord. Becomes tension $T$ pulling on tetherball
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ALSO: Work $= T \cdot \Delta R \neq 0$ has an inward radial part
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Now, the same tetherball is allowed to rotate freely, winding around the pole.

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Discussion of Tetherball 2

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Top view
Discussion of Tetherball 2

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Tension is NOT parallel to radial distance → external torque on the ball.

\[ \vec{r}_{\text{ext}} = \vec{R} \times \vec{T} \neq 0 \]
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→ With Work.
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$\rightarrow$ With Work

Ball direction is always at 90° to tension because of contact point
Work = $T \cdot \Delta R = 0$
A solid disk of mass $m_1 = 9.3 \text{ kg}$ and radius $R = 0.23 \text{ m}$ is rotating with a constant angular velocity of $\omega = 32 \text{ rad/s}$. A thin rectangular rod with mass $m_2 = 3.2 \text{ kg}$ and length $L = 2R = 0.46 \text{ m}$ begins at rest above the disk and is dropped on the disk where it begins to spin with the disk.

1) What is the initial angular momentum of the rod and disk system?

2) What is the initial rotational energy of the rod and disk system?

3) What is the final angular velocity of the disk?

4) What is the final angular momentum of the rod and disk system?

5) What is the final rotational energy of the rod and disk system?

6) The rod took $t = 7 \text{ s}$ to accelerate to its final angular speed with the disk, what average torque was exerted on it by the disk?